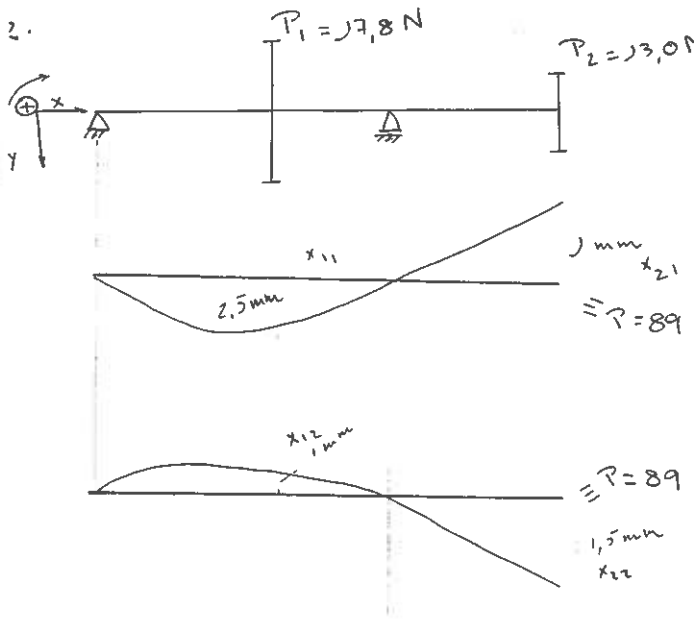
2.



$g = 10\ m/s^2$

Exame 14/1/2005

a)

$$a_{11} = \frac{x_{11}}{P} = \frac{2,5}{17,8}\cdot 10^{-3} = 2,81\cdot 10^{-5}$$

$$a_{21} = \frac{x_{21}}{P} = \frac{-1,0}{17,8}\cdot 10^{-3} = -1,12\cdot 10^{-5}$$

$$a_{12} = \frac{x_{12}}{P} = \frac{-1,0}{13,0}\cdot 10^{-3} = -1,12\cdot 10^{-5}$$

$$a_{22} = \frac{x_{22}}{P} = \frac{1,5}{13,0}\cdot 10^{-3} = 1,69\cdot 10^{-5}$$

$$[a] = \begin{bmatrix} 2,81\cdot 10^{-5} & -1,12\cdot 10^{-5} \\ -1,12\cdot 10^{-5} & 1,69\cdot 10^{-5} \end{bmatrix}$$ matriz de flexibilidade

$$[k] = [a]^{-1} = \frac{1}{|a|}\,\mathrm{cof}^T[a] = \frac{1}{3,47\cdot 10^{-10}}\cdot\begin{bmatrix} 1,69\cdot 10^{-5} & +1,12\cdot 10^{-5} \\ +1,12\cdot 10^{-5} & 2,81\cdot 10^{-5} \end{bmatrix}$$

$$|a| = 3,47\cdot 10^{-10}$$

$$[k] = \begin{bmatrix} 48,55 & +32,36 \\ +32,36 & 80,91 \end{bmatrix}\cdot 10^3\ N/m = \begin{bmatrix} 48,55 & +32,36 \\ +32,36 & 80,91 \end{bmatrix}\ kN/m$$

b) $$[m]\{\ddot{x}\} + [k]\{x\} = 0 \iff \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 80,91 & -32,36 \\ -32,36 & 48,55 \end{bmatrix}10^3\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0$$

c) $$|[k] - \omega^2[m]| = 0$$

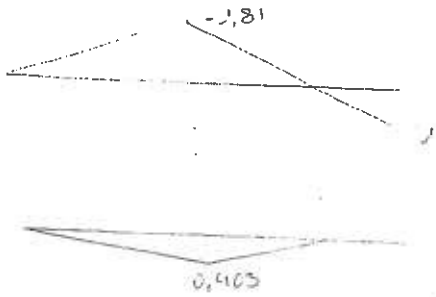
$$\begin{vmatrix} 80,91\cdot 10^3 - 1,78\omega^2 & -32,36\cdot 10^3 \\ -32,36\cdot 10^3 & 48,55\cdot 10^3 - 1,3\cdot\omega^2 \end{vmatrix} = 0 \iff \begin{vmatrix} \omega_1^2 = 1,69\cdot 10^4 \\ \omega_2^2 = 7,04\cdot 10^4 \end{vmatrix} \Rightarrow \begin{vmatrix} \omega_1 = 130\ rad/s \\ \omega_2 = 265,4\ rad/s \end{vmatrix}$$

$$[[k] - \omega_i^2[m]]\{u\}_i = 0$$

$$\begin{bmatrix} 17697 & 32050 \\ 32050 & 58016 \end{bmatrix}\begin{Bmatrix} u_{11} \\ u_{21} \end{Bmatrix} = 0 \qquad \frac{u_{11}}{u_{21}} = -1,81 \qquad \{u\}_1 = \begin{Bmatrix} -1,81 \\ 1 \end{Bmatrix}$$

$$\begin{bmatrix} -79445 & 32050 \\ 32050 & -12930 \end{bmatrix}\begin{Bmatrix} u_{12} \\ u_{22} \end{Bmatrix} = 0 \qquad \frac{u_{12}}{u_{22}} = 0,403 \qquad \{u\}_2 = \begin{Bmatrix} 0,403 \\ 1 \end{Bmatrix}$$

d



e)

$$V = \frac{1}{2}[k]|u|^2$$

f) $$|\phi| = \frac{1}{\sqrt{|u|^T[m]|u|}}|u|$$

$$|\phi|_1 = \begin{vmatrix} -0,67 \\ 0,37 \end{vmatrix} \qquad |\phi|_2 = \begin{vmatrix} 0,32 \\ 0,79 \end{vmatrix}$$

$$[I]|\ddot{\eta}| + [\Omega^2]|\eta| = [\Phi]^T|F|$$

$$\begin{cases} m_1\ddot{\eta}_1 + \omega_1^2\eta = 0 \\ m_2\ddot{\eta}_2 + \omega_2^2\eta = 0 \end{cases}$$

$$\Theta(\omega) = \frac{F/k_1}{\sqrt{(1-\beta^2)^2 + (2\beta\beta)^2}} = \frac{(0,12 + 0,5/2)F/2}{\sqrt{(1-0,89^2)^2 + (2\,0,114\cdot 0,89)^2}}$$

$\Theta(\omega) = 0,64F \quad = 63,7\,\%\,F$

g) $\theta(t) = \Theta(\omega)\cos(\omega t - \phi) = 0,64F\cos(4,33t - 3\pi/4)$

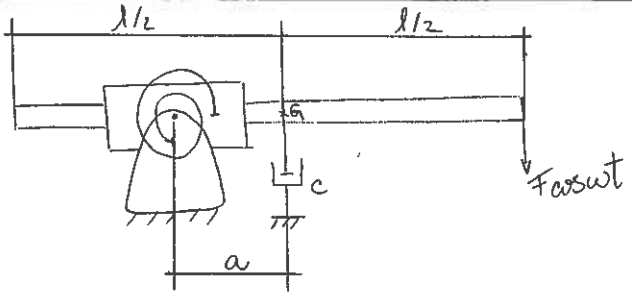
$\dot{\theta}(t) = -4,33\cdot 0,64F\,\mathrm{sen}(4,33t - 3\pi/4) = -2,77F\cos(4,33t - 3\pi/4)$

$f_c = a_c\dot{\theta} = 0,12\cdot 0,0936\cdot 2,77F\cos(4,33t - 3\pi/4)$ sentido oposto ao movimento.

$f_c = 0,031F\cos(4,33t - 3\pi/4)$

Exame 14/1/2005

1.



$l = 0{,}5\,m$

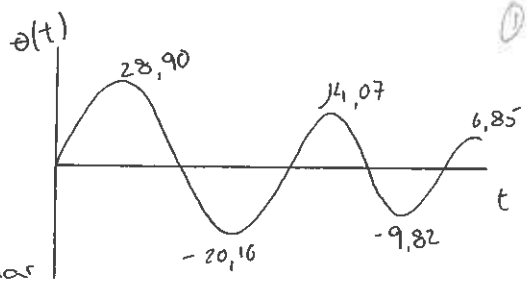
$m_b = 1{,}25\,kg$

$k_t = 2\,Nm/rad$

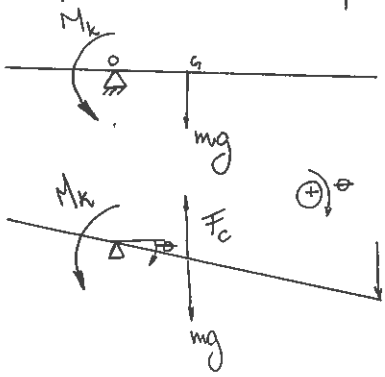
$a = 0{,}12\,m$

$T_d = 1{,}3\,s$

I_o^c - m. inércia colar



a) Na posição de equilíbrio



$\Sigma M_O = 0 \quad M_k = mga \iff k_t \cdot \theta_{est} = mga$ (1)

$\overrightarrow{OG} = \begin{vmatrix} a\cos\theta \\ a\,\text{sen}\,\theta \\ 0 \end{vmatrix}$; $\overrightarrow{\dot{OG}} = \begin{vmatrix} -a\dot\theta\,\text{sen}\,\theta \\ a\dot\theta\cos\theta \end{vmatrix}$

$F_c = c \cdot \dot{\overrightarrow{OG}}_Y = a\dot\theta\cos\theta \cdot c \simeq ac\dot\theta$

$\Sigma M_O = J_O^T \cdot \ddot\theta$; $J_O^T = J_O^c + J_O^b = I_O^c + \frac{1}{12} m_b l^2 + m_b a^2 \iff$

$\iff -M_k - F_c \cdot a\cos\theta' + mg\,a\cos\theta' + (a+\frac{l}{2})\cos\theta' \cdot F\cos\omega t = J_O^T \theta$

$\iff -k_t(\theta + \theta_{est}) - a^2 c\dot\theta + mga + (a+\frac{l}{2}) F\cos\omega t = J_O^T \ddot\theta$

$\iff -k_t\theta - k_t\theta_{est} - a^2 c\dot\theta + mga + (a+\frac{l}{2}) F\cos\omega t = J_O^T \ddot\theta$

(por causa de (1))

$\iff (I_O^c + \frac{1}{12} m_b l^2 + m_b a^2)\ddot\theta + a^2 c\dot\theta + k_t\theta = (a+\frac{l}{2}) F\cos\omega t$

b)

$\delta = \frac{1}{N} \ln \frac{x(t_1)}{x(t_2 = t_1 + NT)} = \ln \frac{28{,}90}{14{,}07} = 0{,}72$

$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = \frac{0{,}72}{\sqrt{4\pi^2 + 0{,}72^2}} = 0{,}114$

c)

$\omega_d = \omega_n \sqrt{1 - \xi^2} \iff \omega_n = \frac{2\pi/1.3}{\sqrt{1 - 0{,}114^2}} = 4{,}86\ rad/s$

d)

$\omega_n = \sqrt{\frac{k_t}{m}} \Rightarrow m = \frac{T_O^T}{} = \frac{k_t}{\omega_n^2} = \frac{2}{4{,}86^2} = 0{,}0845$

e)

$c = \xi\, 2m \cdot \omega_n = 2 \cdot 0{,}114 \cdot 0{,}0845 \cdot 4{,}86 = 0{,}0936\ kg/s$

f)

$\phi = 135°$ $\quad \phi = tg^{-1} \frac{2\xi\beta}{1-\beta^2}$ $\quad tg\left(135 \cdot \frac{\pi}{180}\right) = \frac{2 \cdot 0{,}114\beta}{1-\beta^2} = -1$

$1 - \beta^2 = 0{,}228\beta$

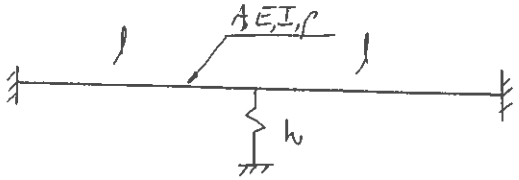
$\beta^2 + 0{,}228\beta - 1 = 0$

$\beta = \begin{vmatrix} \beta_1 = -1{,}12 \ X \\ \beta_2 = 0{,}89 \end{vmatrix}$ $\quad \beta = \frac{\omega}{\omega_n} \Rightarrow \omega = 0{,}89 \cdot 4{,}86$

$\omega = 4{,}33\ rad/s$

3.

Exame 14/1/2005



Se $h=0$  da pag. 380 da sebenta sei q̇

$\cos\beta l \cosh\beta l = 1$; $\omega_i = (\beta l)_i^2 \sqrt{\frac{EI}{\rho A l^4}}$

Para a 1º freq nat de vibração $\beta l = 4{,}73$

$$\omega_1 = 4{,}73^2 \sqrt{\frac{EI}{\rho A l^4}}$$

Se $h=\infty$   da mesma tabela obtemos:

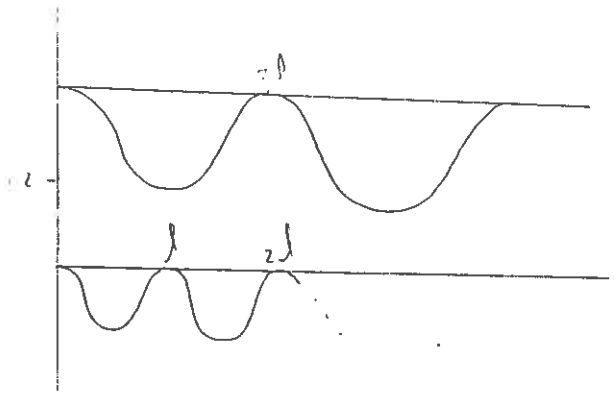
$\beta l = 3{,}93$ | $\beta l = 1{,}875$

$\omega = 3{,}93^2 \sqrt{\frac{EI}{\rho A l^4}}$ | $\omega = 1{,}875^2 \sqrt{\frac{EI}{\rho A l^4}}$

$$\omega_{total} = (3{,}93^2 + 1{,}875^2)\sqrt{\frac{EI}{\rho A l^4}}$$

b) $\psi_1(x) = 1 \cdot \cos\left(\frac{\pi}{l}x\right)$

$\psi_2(x) = 1 \cdot \cos\left(\frac{2\pi}{l}x\right)$



Condições de Fronteira: